UNEMPLOYMENT AND INPUT PRICES:
A FRACTIONAL COINTEGRATION APPROACH

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This paper examines the relationship between unemployment, real oil price and real interest rates in Canada. Instead of following the classical approach based on I(0) stationarity or I(1) cointegrating relationships, we use fractional integration/cointegration techniques which allow for the possibility that unemployment is highly persistent. In line with other studies, we find that all three variables are I(1). But we only find cointegration in the presence of autocorrelated disturbances, which means that the relationship between these variables also has a dynamic component. Furthermore, there is evidence of fractional (as opposed to classical) cointegration, which implies long memory and slow reversion to equilibrium. This suggests that an equilibrium model with highly persistent shocks might be adequate to account for the observed behaviour of unemployment.

Keywords: *Unemployment, Input Prices, Long Memory, Fractional Integration, Fractional Cointegration*

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1. Introduction

A number of recent studies have emphasised the role of real oil prices and real interest rates as primary determinants of unemployment (see, e.g., Bean, 1994, Phelps, 1994, Nickell, 1997, 1998, Blanchard, 1999). In particular, Carruth et al (1998) have argued that the main movements in US postwar unemployment are well explained in the context of an efficiency-wage model in which input prices affect the equilibrium rate. In their model increases in input prices result in a fall in wages, because of the zero-profit condition in the product market, which implies that unemployment must go up for workers to accept lower wages. Specifically, they consider two input prices (namely, the real price of oil and the real rate of interest), and examine their empirical relationship with unemployment in the US by carrying out Granger causality tests and estimating an error-correction model (ECM). They conclude that both sets of results are consistent with their model, which appears to account satisfactorily for the past behaviour of unemployment in the US, and also to perform well in terms of out-of-sample forecasts.

In this paper, we adopt the same theoretical framework to explain the Canadian experience. However, we take a different modelling approach. Instead of following the classical approach based on I(0) stationarity or I(1) cointegrating relationships, we use fractional integration/cointegration techniques which allow for the possibility that unemployment is highly persistent (see Robinson, 1994, and Caporale and Gil-Alana, 2000a). In line with other studies, we find that all three variables are I(1). But we only find cointegration in the presence of autocorrelated disturbances, which means that the relationship between these variables also has a dynamic component. Furthermore, there is evidence of fractional (as opposed to classical) cointegration, which implies long memory and slow reversion to equilibrium. This suggests that an equilibrium model with highly persistent shocks might be adequate to account for the observed behaviour of unemployment.

The paper is structured as follows. Section 2 outlines the testing strategy we employ. Section 3 describes the data and presents the empirical results. Section 4 offers some concluding remarks.

2. Testing for fractional integration and cointegration

In studies relying on standard cointegration analysis the equilibrium errors are restricted to be an I(0) process, which is not persistent. However, it might be the case that the equilibrium errors respond more slowly to shocks, which results in highly persistent deviations from equilibrium. Therefore, we describe below a testing procedure which allows for the possibility of a long-memory cointegrating relationship, and which could enable us to gain a better understanding of the relationship between unemployment and input prices (see Caporale and Gil-Alana, 2000a, for more details and some Monte Carlo evidence on the power and size properties of the suggested test).

For the purpose of the present paper, we define an I(0) process u_t, t = 0, ±1, ….., as a covariance stationary process with spectral density which is positive and finite at zero frequency. In this context, an I(d) process, x_t, t = 0, ±1, …., is defined by

$$(1-L)^d\, x_t \;=\; u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

$$x_t = 0 \qquad t \le 0 \tag{2}$$

where L is the lag operator and u_t is I(0). The macroeconomic literature focuses on the cases d = 0 and d = 1 (see, e.g., Nelson and Plosser, 1982), whereas we define $(1-L)^d$ for all real d by

$$1 \;+\; \sum_{j=1}^{\infty} \frac{\Gamma(d+1)(-L)^j}{\Gamma(d-j+1)\;\Gamma(j+1)} \;=\; 1 \;-\; d\,L \;+\; \frac{d\;(d-1)}{2} L^2 \;-\; \frac{d\;(d-1)\;(d-2)}{3!}\, L^3 \;+\; \ldots\;.$$

The process u_t in (1) could be a stationary and invertible ARMA sequence, with an exponentially decaying autocovariance function. This property can be said to characterise a "weakly autocorrelated" process. When d = 0, $x_t = u_t$, so a "weakly autocorrelated" x_t is allowed for. When d = 1, x_t has a unit root, while for a general integer d, x_t has d unit roots. For $0 < d < 0.5$, x_t is still stationary, but its lag-j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \to \infty$, and so the γ_j are non-summable. The distinction between I(d) processes with different values of d is also important from an economic point of view: if a variable is an I(d) process with $d \in [0.5, 1)$, it will be covariance nonstationary but mean-reverting since an innovation will have no permanent effect on its value. This is in contrast to an I(1) process which will be both covariance nonstationary and non-mean-reverting, in which case the effect of an innovation will persist forever.

Robinson (1994) proposes LM tests for testing unit roots and other forms of nonstationary hypotheses, embedded in fractional alternatives. A very simple version of his tests consists in testing the null hypothesis:

$$H_o:\ \theta = 0 \qquad (3)$$

in a model given by

$$(1 - L)^{d+\theta} x_t = u_t, \qquad t = 1, 2, \dots , \qquad (4)$$

where x_t is the time series we observe; u_t is an I(0) process, and d is a given value that may be 1 but also any other real number. Specifically, the score test statistic proposed by Robinson (1994) takes the form:

$$\hat{r} = \left(\frac{T^{1/2}}{\hat{\sigma}^2}\right) \hat{A}^{-1/2}\, \hat{a}, \qquad (5)$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1}\, I(\lambda_j)$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j) \right)$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L)^d x_t$, evaluated at $\lambda_j = 2\pi j/T$, and g is a known function coming from the spectral density function of $\hat{u}_t$: $f(\lambda_j;\tau) = \frac{\sigma^2}{2\pi} g(\lambda_j;\tau)$, with $\hat{\tau}$ obtained by minimising $\sigma^2(\tau)$. Robinson (1994) showed that under certain regularity conditions:

$$\hat{r} \to_d N(0,1) \qquad as \qquad T \to \infty. \tag{6}$$

Thus, a one-sided 100α%-level test of (3) against the alternative H_1: $\theta > 0$ is given by the rule: 'Reject H_o if $\hat{r} > z_\alpha$', where the probability that a standard normal variate exceeds z_α is α, and, conversely, an approximate one-sided 100α%-level test of (3) against the alternative H_1: $\theta < 0$ is given by the rule: 'Reject H_o if $\hat{r} < -z_\alpha$'. Furthermore, he shows that the above tests are efficient in the Pitman sense, i.e. that against local alternatives of the form: H_a: $\theta = \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic.[1]

Having defined fractional integration and described a way of testing I(d) statistical models, next we introduce the concept of fractional cointegration. By adopting the simplest possible definition, it can be said that a given vector X_t is fractionally cointegrated if:

a) all its components (X_{it}) are integrated of the same order (say d), i.e.,

$$(1-L)^d x_{it} = u_{it}, \quad t = 1,2,..., \ for\ all\ i,$$

[1] An empirical application of this testing procedure using historical U.S. annual data can be found in Gil-Alana and Robinson (1997) and other versions of the tests based on seasonal (quarterly and monthly) and cyclical data are respectively Gil-Alana and Robinson (2000) and Gil-Alana (1999,2000).

with I(0) u_i's, and

b) there is at least one linear combination of these components which is fractionally integrated of order b, with b < d.[2]

We propose here a two-step procedure for testing the null hypothesis of no cointegration against the alternative of fractional cointegration, which is based on the univariate tests of Robinson (1994). In the first step, Robinson's (1994) tests are used to test the order of integration of each of the individual series and, if all are integrated of the same order (say d), as a second step, the degree of integration in the residuals from the cointegrating regression is tested. A difficulty emerges here in that the residuals are not actually observed but obtained from the cointegrating regression, and thus there might be a bias in favour of stationary residuals. Note that this problem is similar to the one encountered by Engle and Granger (1987) when testing cointegration with the tests of Fuller (1976) and Dickey and Fuller (1979). In order to solve this problem, finite-sample critical values of the tests were computed in Caporale and Gil-Alana (2000a,b). We can consider the model,

$$(1 - L)^{d+\theta} e_t = v_t, \qquad t = 1, 2, \ldots$$

where e_t are the OLS residuals[3] from the cointegrating regression and I(0) v_t, and test H_o (3) against the one-sided alternative:

$$H_a: \quad \theta < 0. \tag{7}$$

Note that if we cannot reject H_o (3) on the estimated residuals, we will find evidence of no cointegration, since the residuals will be integrated of the same order as the univariate series. On the other hand, rejections of H_o (3) against (7) will provide evidence of

[2] A more general definition of fractional cointegration, allowing different integration orders for each series, can be found for example in Marinucci and Robinson (1998).

[3] In standard cointegration analysis, Stock (1987) showed that the LS estimate of the cointegrating parameter was a consistent estimate of the true value. Cheung and Lai (1993) and others extended the analysis to the case of fractional cointegration and showed that the LS estimate was also consistent in this case, though with possible different convergence rates, depending on the cointegrating vector.

cointegration of a certain degree since the residuals will be integrated of a smaller order than that of the individual series.

3. Data and empirical results

The raw series we use are the total unemployment level, an oil price index (specifically, the industrial price index for refined petroleum and coal products, which is the available series with the longest time span), and the 5-year benchmark government bond yield (end of the month). The real oil price and real interest rates series have been constructed using the GDP deflator. The data are quarterly and seasonally adjusted, and cover the period from 1966q1 to 2000q2. The source is Datastream.

We begin by plotting the data. Figure 1 clearly shows that the three series follow similar patterns over time, implying that they might be linked through a long-run relationship. Next we perform Robinson's (1994) univariate unit root tests on the individual series. The results are reported in Table 1. We find that in practically all cases there is evidence of a unit root at the 95% significance level, regardless of the specification chosen for the disturbances, be they a white noise, AR(1) or AR(2) process. The only exception is unemployment with AR(2) disturbances, but even in this case the presence of a unit root cannot be rejected at the 99% significance level.

Having found that all series exhibit unit root behaviour, we then examine whether they are linked through a long-run relationship. The results from a cointegrating regression of unemployment against both input prices are displayed in Table 2. As can be seen, all coefficients appear to be significantly different from zero and to have signs which can be justified on theoretical grounds (positive for the oil prices and negative for the interest rate). In order to establish whether this is indeed a long-run equilibrium relationship, we need to test for the order of integration of the estimated residuals. Table 3 shows that if the disturbances are white noise there is no evidence of cointegration. By constrast, if they are autocorrelated (AR(1) and AR(2)), there is evidence of fractional cointegration. This leads us to conclude that the relationship under study has both a long- and a short-run component, and also exhibits a high degree of persistence, such that deviations from equilibrium are long-lived. Consequently, an equilibrium model combined with a limited set of shocks with extreme persistence can capture well the behaviour of actual unemployment.

4. Conclusions

In this paper we have examined the relationship between unemployment and input prices in Canada using the theoretical framework suggested by Carruth et al (1998), which assigns a role to both real oil prices and real interest rates in the determination of equilibrium unemployment.[4] We have argued that the discrete options I(1) and I(0) offered by classical cointegration analysis are rather restrictive. Adjustment to equilibrium might in fact take a longer time than suggested by standard cointegration tests. In other words, unemployment and input prices might be tied together through a fractionally integrated I(d)-type process such that the equilibrium errors exhibit slow mean reversion. In such a case, although there exists a long-run relationship, the error correction term possesses long memory, and hence deviations from equilibrium are highly persistent.

Our empirical results can be interpreted as indicating that the possible relationship linking Canadian unemployment and input prices involves both a long-run component (given by the static relationship which has been estimated) and a short-run component, which is represented by the autoregressive process followed by the disturbances.

[4] In efficiency-wage models, the effects of the oil price are generally modelled as temporary (see, e.g., Bruno and Sachs, 1982, and Hamilton, 1988), or only indirectly (see Layard and Nickell, 1991), or even this variable is not treated as an input (see, e.g. Phelps, 1994). Nickell (1990) and Phelps (1994) both highlight the possible role of interest rates.

Moreover, the adjustment towards equilibrium takes a long time. Therefore, although it does appear that input prices play an important role in driving the long swings observed in unemployment (as suggested, e.g., by Phelps, 1994, Nickell, 1997, 1998, Blanchard, 1999, and Carruth et al, 1998), the relationship is one which involves the dynamics as well, and is characterised by prolonged persistence.[5] As a result, the effects of shocks will take a long time to disappear.

A NAIRU (Non Accelerating Inflation Rates of Unemployment) model with a limited set of highly persistent shocks might therefore provide a better explanation for the observed behaviour of unemployment than a model with hysteresis focusing on movements in the equilibrium rate when accounting for movements in the actual rate (see, e.g., Blanchard and Summers, 1986), with important policy implications.

[5] As models of the labour market with "rigidities" would imply.

References

Bean, C.R. (1994), "European unemployment: a survey", *Journal of Economic Literature*, 32, 573-619.

Blanchard, O. and L. Summers (1986), "Hysteresis and the European unemployment problem", *NBER Macroeconomic Annual,* 1, 15-78.

Blanchard, O. (1999), "Revisiting European unemployment: unemployment, capital accumulation and factor prices", *NBER WP no. 6566.*

Bruno, M. and J. Sachs (1982), "Input price shocks and the slowdown of economic growth: the case of UK manifacturing", *Review of Economic Studies*, 49, 679-705.

Caporale, G.M. and L.A. Gil-Alana (2000a), "Fractional cointegration and tests of present value models", *D.P. no. 15, Quantification and Simulation of Economic Processes, Humboldt Universitat zu Berlin.*

Caporale, G.M. and L.A. Gil-Alana (2000b), "Fractional cointegration and aggregate money demand functions", *D.P. no. 03-00, Centre for Monetary and Financial Economics, South Bank University, London.*

Carruth, A.A., Hooker, M.A. and A.J. Oswald (1998), "Unemployment equilibria and input prices: theory and evidence from the United States", *Review of Economics and Statistics,* 80, 4, 621-628.

Cheung Y.W. and K.S. Lai (1993), "A fractional cointegration analysis of purchasing power parity", *Journal of Business and Economic Statistics*, 11, 103-112.

Dickey, D.A. and W.A. Fuller (1979), "Distribution of the estimators for autoregressive time series with a unit root", *Journal of the American Statistical Association*, 74, 427-431.

Dickey, D.A. and W.A. Fuller (1981), "Likelihood ratio statistics for autoregressive time series with a unit root", *Econometrica*, 49, 1057-1072.

Engle, R.F. and C.W.J. Granger (1987), "Co-integration and error correction: representation, estimation, and testing", *Econometrica*, 55, 2, 251-276.

Fuller, W.A. (1976), *Introduction to Statistical Time Series*, New York, Wiley.

Gil-Alana, L.A. (1997), "Testing fractional integration in macroeconomic time series", *Ph.D. thesis, London School of Economics and Political Sciences.*

Gil-Alana, L.A. (1999), "Testing fractional integration with monthly data", *Economic Modelling,* 16, 613-629.

Gil-Alana, L.A. (2000), "Testing stochastic cycles in macroeconomic time series", forthcoming in *Journal of Time Series Analysis*.

Gil-Alana, L.A. and P.M. Robinson (1997), "Testing of unit root and other nonstationary hypotheses in macroeconomic time series", *Journal of Econometrics*, 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson (2000), "Testing seasonal fractional integration in the UK and Japanese consumption and income", forthcoming in *Journal of Applied Econometrics*.

Hamilton, J.D. (1988), "A neoclassical model of unemployment and the business cycle", *Journal of Political Economy*, 96, 593-617.

Layard, R., Nickell, S.J. and R. Jackman (1991), *Unemployment: Macroeconomic Performance and the Labour Market,* Oxford University Press, Oxford.

Marinucci, D. and P.M. Robinson (1998), "Semi-parametric frequency domain analysis of fractional cointegration", *STICERD, London School of Economics, Discussion Paper Series, EM/98/348.*

Nelson, C.R. and C.I. Plosser (1982), "Trends and random walks in macroeconomic time series", *Journal of Monetary Economics*,10, 139-162.

Nickell, S.J. (1990), "Unemployment: a survey", *Economic Journal*, 100, 391-439.

Nickell, S.J. (1997), "Unemployment and labour market rigidities: Europe versus North America", *Journal of Economic Perspectives*, 11, 3, 55-74.

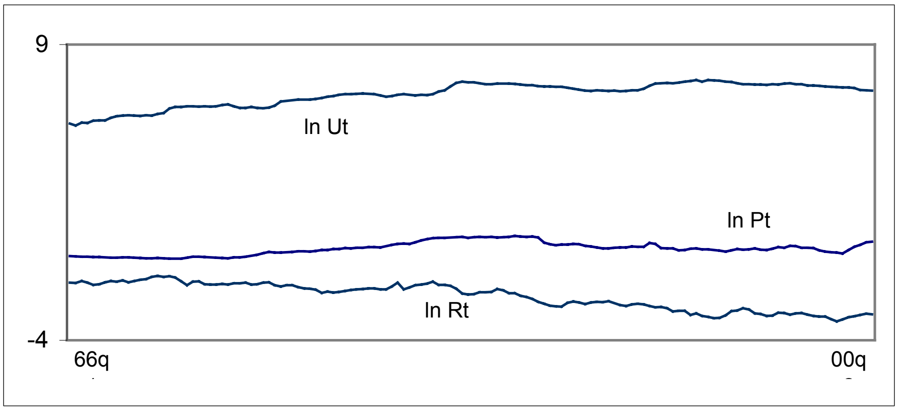
Nickell, S.J. (1998), "Unemployment: questions and some answers", *Economic Journal*, 108, 802-816.

Phelps, E.S. (1994), *Structural Slumps*, Harvard University Press, Cambridge. Mass.

Robinson, P.M. (1994), "Efficient tests of nonstationary hypotheses", *Journal of the American Statistical Association*, 89, 1420-1437.

Stock, J.H. (1987), "Asymptotic properties of least squares estimators of cointegrating vectors", *Econometrica*, 55, 1035-1056.

FIGURE 1



Ln Ut, Ln Pt and Ln Rt means respectively the logarithm transformations of the unemployment, the real oil prices and the real interest rate series.

TABLE 1			
Unit root testing using the tests of Robinson (1994)			
Series / Disturbances	White noise	AR (1)	AR (2)
UNEMPLOYMENT	**-0.50’**	**0.77’**	-2.08
REAL OIL PRICES	**1.13’**	**-1.54’**	**0.41’**
REAL INTEREST RATES	**-0.73’**	**-1.61’**	**-1.43’**

‘ and in bold: Non-rejection values of the null hypothesis of a unit root at the 95% significance level.

TABLE 2			
OLS regression of unemployment on oil prices and interest rates			
	Intercept	Oil prices	Interest rates
Coefficient	5.620	0.987	-0.555
Standard Error	0.064	0.060	0.030

TABLE 3											
Testing the order of integration of the residuals from the cointegrating regression											
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
W.N.	19.57	18.28	16.77	15.01	13.04	10.90	8.69	6.51	4.48	2.66	1.09’
AR (1)	2.88	**1.02’**	**0.25’**	**-0.34’**	**-0.71’**	**-0.81’**	**-1.68’**	**-1.97**	**-2.12’**	-2.33	-2.50
AR (2)	11.08	11.04	9.74	6.89	4.44	2.59	**1.21’**	**-0.21’**	**-0.46’**	**-1.86’**	-2.83

‘ and in bold: Non-rejection values of the null hypothesis of a unit root at the 95% significance level.